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DYNAMICS AND CONTROL OF FLEXIBLE SPACECRAFT
DURING AND AFTER SLEWING MANEUVERS

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INTRODUCTION

Many future NASA missions would utilize significantly large and flexible spacecrafts and would require very stringent pointing and vibration suppression requirements. The active controller that can achieve these objectives will have to be designed with very accurate knowledge of the dynamic behavior of the spacecraft to ensure performance robustness to a variety of disturbances and uncertainties.

In the past few years, several design approaches were proposed for vibration control during and after slewing maneuvers. NASA Langley Research Center initiated the Spacecraft Control Laboratory Experiment (SCOLE) program [1] to promote direct comparison and realistic test of various control design techniques against a common laboratory article. The article was intended to resemble a large space antenna attached to the space shuttle orbiter by a long flexible mast.

The primary control objective of SCOLE is to direct the RF line-of-sight (LOS) of the antenna-like configuration towards a fixed target under conditions of minimum time and limited control authority.

This problem of directing the LOS of antenna-like configuration is studied as being composed of two-phase control problem during the current research period. The two phases are namely, the slew maneuver control of rigidized-body of SCOLE configuration and the vibration suppression of flexible antenna. This formulation allows the design of control systems using a decentralized control scheme in which the dynamics of the two phases of control problem are viewed as two subsystems with some interaction. The residual

vibration suppression of the flexible antenna at the end of the slew maneuver is viewed as the second phase of the control problem.

BRIEF SUMMARY OF MAJOR ACCOMPLISHMENTS

(a) Decentralized Slew Maneuver of SCOLE

During the current research period, the basic software for the decentralized slew maneuver control of SCOLE model was completed. The software was based on an algorithm of Hierarchical Optimal Control formulation in which the SCOLE model is viewed to be a large-scale dynamical system composed of interconnected subsystems. It was deduced that the system could naturally be decomposed along the lines of dynamics of rigid part and those of flexible antenna. Thus the set of uncoupled subsystems was generated together with their coupling relations. The performance index to be minimized was written in terms of the two subsystems and the subsystems were further formulated individually in terms of two-point boundary value problems. The necessary conditions for the individual subsystems were developed in terms of nonlinear differential equations for optimal control schemes.

(b) Quasilinearization of Subsystems:

The two-point boundary value problem of each subsystem was solved by the method of quasilinearization. The process of quasilinearization for each subsystem involved the linearization of the nonlinear system equations and utilizing the method of complementary functions. An iterative search was incorporated in the algorithm to get the final set of equations which satisfy the split boundary conditions.

The complete control system design was developed at a second-level in terms of a coordinating algorithm to optimize the overall state trajectory.

The development of the software to design an optimal state feedback control system for each subsystem allows specifications of arbitrary large-angle nonlinear slew maneuvers. Computer simulations of second-level trajectory optimizations were obtained and are being analyzed for determining global optimal solutions.

(c) Shooting Method

The two-point boundary value problem for each subsystem is also being solved using shooting method instead of quasilinearization method to compare the solutions in terms of numerical convergence.

(d) Presentations and Publications

A detailed technical report based on Slew Maneuver Control has been prepared for publication as NASA Contractor Report. A presentation entitled, "Decentralized Slew Maneuver Control and Vibration Suppression of Large Flexible Spacecrafts," was presented at the Workshop on Computational Techniques in Identification and Control of Flexible Flight Structures, Lake Arrowhead, California, November, 1989. This paper is also to be published in the proceedings. A second paper entitled, "Slew Maneuvers of Large Flexible Spacecrafts" is to be presented and published at the 1990 American Control Conference in May 1989.

A chapter based on collective work of previous years is to be published in a book entitled, "Modelling and Control of Large Space Structures: The SCOLE Experience" to be edited by A. V. Balakrishnan and L. Taylor. This is included in the appendix.

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APPENDIX

SLEW MANEUVER DYNAMICS AND CONTROL OF SPACECRAFT CONTROL LABORATORY EXPERIMENT (SCOLE)

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ABSTRACT

In this article, the dynamics and control of slewing maneuvers of NASA Spacecraft Control Laboratory Experiment (SCOLE) are analyzed. The control problem of slewing maneuvers of SCOLE is formulated in terms of an arbitrary maneuver about any given axis. The control system is developed for the combined problem of rigid-body slew maneuver and vibration suppression of the flexible appendage. The control problem formulation incorporates the nonlinear dynamical equations derived previously in a report [1] and is expressed in terms of a two-point boundary value problem utilizing a quadratic type of performance index.

The two-point boundary value problem is solved as a hierarchical control problem with the overall system being split in terms of two subsystems, namely the slewing of the entire assembly and the vibration suppression of the flexible antenna. The coupling variables between the two dynamical subsystems are identified and these two subsystems for control purposes are treated independently in parallel at the first level. Then the state-space trajectory of the combined problem is optimized at the second level.

1. INTRODUCTION

The primary control objective of the Spacecraft Control Laboratory Experiment (SCOLE) is to direct the RF Line-Of-Sight (LOS) of the antenna-like configuration towards a fixed target under the conditions of minimum time and limited control authority [2]. This problem of directing the LOS of antenna-like configuration involves both the slewing maneuver of the entire assembly and the vibration suppression of the flexible antenna-like beam. The study of ordinary rigid-body slew maneuvers has received considerable attention in the literature [3,4] due to the fact that any arbitrary large-angle slew maneuver involves kinematic nonlinearities. This is further complicated in the case of SCOLE by virtue of a flexible appendage deployed from the rigid space shuttle. The dynamics of arbitrary large-angle slew maneuvers of SCOLE model were derived previously by the author in a report [1] as a set of coupled equations with the rigid-body motions including the nonlinear kinematics and the vibratory equations of the flexible appendage. These nonlinear and coupled dynamical equations are used in this article to study the slew maneuver control in terms of a hierarchical feedback control scheme.

The control problem of slewing maneuvers of this large flexible spacecraft is developed by using the two-point boundary value problem in terms of the rigid-body slewing and the vibration suppression of the flexible appendage as two separate dynamical subsystems. A decentralized optimal control scheme is utilized in order to solve individual boundary-value problem for each of the two subsystems by defining their state variable models and incorporating the coupling variables between the two subsystems in these models. Also, the boundary conditions of the overall system are reworked in terms of boundary conditions of each subsystem. A quadratic performance index is utilized for the overall system and is subsequently expressed in terms of a sum of two individual performance indices of

the subsystems.

The basic algorithm for obtaining an optimal closed-loop state feedback scheme involves using a trajectory in terms of a vector of Lagrange multipliers as an initial guess at level two. This is used at level one in quasilinearization application.

The two-point boundary value problem for each subsystem is solved at level one by using a quasilinearization technique as a trajectory optimization problem. In the quasilinearization procedure, starting from an initial guessed state trajectory, successive linearizations are performed of state equations in such a way that the final solution of the state trajectory is within an acceptable degree subject to the boundary conditions. The state vector definition at this level is an augmented state vector which includes both system states and costates.

These optimum solutions of the subsystem trajectories are utilized at level two to yield the updated trajectory of the vector of Lagrange multipliers of the overall system to be used for quasilinearization process at level one. The basic steps of the algorithm are repeated to optimize this second level trajectory with respect to a prespecified error criterion to obtain an optimal feedback law.

2. LIST OF SYMBOLS

\underline{a}	Vector to the point of force application on the beam
B	Damping matrix
$\underline{E}_2(t)$	Force applied at the reflector mass center
$\underline{G}_o(t)$	Moment applied about the orbiter mass center
I_o	Equivalent Mass moment of inertia of total assembly
I_2	Mass moment of inertia matrix of the reflector
J	Functional used for two-point boundary value problem
K	The stiffness matrix
L	The Length of the beam
M	Angular velocity vector transformation
M_ψ	Effective moment applied at the reflector c.g.
N	The total number of subsystems
\underline{Q}	The generalized force vector
q_i	Generalized coordinates
\underline{r}	Position vector from the orbiter mass center to the point of attachment
r_x	x co-ordinate of the reflector mass center in the body-fixed frame
r_y	y co-ordinate of the reflector mass center in the body-fixed frame
\underline{u}_i	Control vector of i th system
\underline{x}_i	State vector of i th system
\underline{z}_i	Vector of interconnecting variables
$\underline{\gamma}$	Unit vector representing the axis of rotation during the slew maneuver
ϕ_{xi}	i th Eigenfunction corresponding to u_x
ϕ_{yi}	i th Eigenfunction corresponding to u_y

$\phi_{\psi i}$	i th Eigenfunction corresponding to u_{ψ}
$\underline{\theta}$	The attitude of the orbiter in the inertial frame
ξ	Slew Angle
$\underline{\omega}$	The angular velocity of the orbiter in the inertial frame
$\underline{\Omega}$	The angular velocity of the reflector in the inertial frame
$\underline{\epsilon}$	Vector of Euler parameters
$\delta(z-z_j)$	Dirac delta function
$\Phi(\underline{\lambda})$	Dual functional for two-point boundary value problem
$\underline{\lambda}$	Vector of Lagrange multipliers

3. ANALYTICS

Slew Maneuver Specification and Control Variables

The analytics for the dynamics of SCOLE developed in reference [1] are used to derive the control laws for an arbitrary slew maneuver. It is assumed that the slew maneuver is performed by applying moments on the rigid shuttle and the vibration suppression is achieved by means of forces on the flexible antenna and the reflector. The slew maneuver is considered to be an arbitrary maneuver about any given axis [1].

The slew maneuver is defined in terms of $\underline{\gamma}$, the axis about which the slew maneuver is performed. If ξ is the slew angle and $\underline{\omega}$ is the angular velocity of the orbiter in the inertial frame, then the four Euler parameters can be defined as

$$\begin{aligned}\epsilon_1 &= \gamma_1 \sin \frac{\xi}{2} \\ \epsilon_2 &= \gamma_2 \sin \frac{\xi}{2} \\ \epsilon_3 &= \gamma_3 \sin \frac{\xi}{2} \\ \epsilon_4 &= \cos \frac{\xi}{2} \quad .\end{aligned}\tag{1}$$

The four Euler parameters can be related to the angular velocity components of the rigid assembly as

$$\begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \\ \dot{\epsilon}_3 \\ \dot{\epsilon}_4 \end{bmatrix} = \begin{bmatrix} \epsilon_1 & \epsilon_4 & -\epsilon_3 & \epsilon_2 \\ \epsilon_2 & \epsilon_3 & \epsilon_4 & -\epsilon_1 \\ \epsilon_3 & -\epsilon_2 & \epsilon_1 & \epsilon_4 \\ \epsilon_4 & -\epsilon_1 & -\epsilon_2 & -\epsilon_3 \end{bmatrix} \begin{bmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} .\tag{2}$$

The slewing maneuver can be given in terms of the following equations [1]

$$I_o \dot{\underline{\omega}} + A \underline{\ddot{q}} = \underline{G}(t) + \underline{N}_2(\underline{\omega})\tag{3}$$

$$A_2^T \dot{\underline{Q}} + A_3 \ddot{\underline{q}} + B \dot{\underline{q}} + K \underline{q} = \underline{Q}(t) \quad (4)$$

where,

$\underline{Q}(t)$ is the net moment applied about the mass center of the orbiter and is given by the following equation (figs. 1 & 2)

$$\underline{Q}(t) = \underline{Q}_o(t) + (\underline{r} + \underline{a}) \times \underline{F}_2 \quad (5)$$

Also, $\underline{Q}(t)$ represents the generalized force vector which is given by the following equation

$$\underline{Q}(t) = \begin{bmatrix} \sum_{j=1}^m (Q_{jx_1}(t) + Q_{jy_1}(t)) + Q_{x_1} + Q_{y_1} + Q_{\psi_1} \\ \sum_{j=1}^m (Q_{jx_2}(t) + Q_{jy_2}(t)) + Q_{x_2} + Q_{y_2} + Q_{\psi_2} \\ \dots \\ \dots \\ \dots \\ \sum_{j=1}^m (Q_{jx_i}(t) + Q_{jy_i}(t)) + Q_{x_i} + Q_{y_i} + Q_{\psi_i} \\ \dots \\ \dots \end{bmatrix} \quad (6)$$

where, the generalized force components are given as

$$Q_{jx_i} = \int_0^L F_{jx}(z, t) \delta(z - z_j) \phi_{xi}(z) dz \quad (7)$$

$$Q_{jy_i} = \int_0^L F_{jy}(z, t) \delta(z - z_j) \phi_{yi}(z) dz \quad (8)$$

and

$$Q_{j\psi_i}(t) = 0 \quad (9)$$

Here, $F_{jx}(z, t)$ is the x component of the concentrated force applied at location j on the flexible antenna and F_{jy} is the y component of that force.

Also,

$$\begin{aligned} Q_{xi}(t) &= F_{2x}(t)\phi_{xi}(L) \\ Q_{yi}(t) &= F_{2y}(t)\phi_{yi}(L) \\ Q_{\psi i}(t) &= M_{\psi}(t)\phi_{\psi i}(L) . \end{aligned} \quad (10)$$

Here, \underline{F}_2 is the force applied at the reflector C. G.

Thus,

$$M_{\psi}(t) = F_{2x}r_y + F_{2y}r_x + M_{2\psi} \quad (11)$$

The location of reflector C. G. is given by coordinates (r_x, r_y) and $M_{2\psi}$ represents the external moment applied at the reflector C. G. Also, the nonlinearities \underline{N}_2 can be expressed in terms of pure rigid body kinematic nonlinearity and the nonlinear coupling term between the rigid-body modes and the flexible modes.

$$\underline{N}_2 = A_4(\underline{\omega}, \underline{\theta}) + A_5(\underline{\omega}, \underline{\theta})\dot{\underline{q}} \quad (12)$$

The details of this term \underline{N}_2 are included in appendix A and the equation (12) can be further simplified in terms of Euler parameters by relationships developed in Appendix B as

$$\underline{N}_2 = A_6(\underline{\omega}, \underline{\epsilon}) + A_7(\underline{\omega}, \underline{\epsilon})\dot{\underline{q}} \quad (13)$$

where $\underline{\epsilon}$ is the Euler vector comprising all four Euler parameters.

From equations (3) and (4) and by defining $A = -A_2^T I_o^{-1} A_2 + A_3$, the following equations are obtained

$$\begin{aligned} \dot{\underline{\omega}} = I_o^{-1} & \left[A_2 A^{-1} B \dot{\underline{q}} + A_2 A^{-1} K \underline{q} + \left\{ A_2 A^{-1} A_2^T I_o^{-1} + I_3 \right\} \underline{G}(t) \right. \\ & \left. + \left\{ A_2 A^{-1} A_2^T I_o^{-1} + I_3 \right\} \underline{N}_2(\underline{\omega}, \underline{\epsilon}) \right] \end{aligned} \quad (14)$$

$$\ddot{\underline{q}} = -A^{-1}B\dot{\underline{q}} - A^{-1}K\underline{q} - A^{-1}A_2^T I_o^{-1}\underline{G}(t) + A^{-1}A_2^T I_o^{-1}\underline{N}_2(\underline{\omega}, \underline{\epsilon}) + A^{-1}\underline{Q}(t) . \quad (15)$$

It is assumed that control forces applied for vibration suppression has negligible effect on rotational maneuver of the spacecraft in developing equations (14) and (15). Also, I_3 represents 3x3 identity matrix in these equations.

Subsystems and State Variable Models

The two dynamical subsystems considered for decentralized control are the dynamics of the slewing of the rigidized SCOLE assembly and the vibration dynamics of the flexible antenna. These subsystems are represented by subscripts I and II respectively for subsequent analysis.

The following are the definitions of state variables and control variables for subsystem I.

$$x_{I1} \triangle \epsilon_1; \quad x_{I2} \triangle \epsilon_2; \quad x_{I3} \triangle \epsilon_3; \quad x_{I4} \triangle \epsilon_4;$$

$$x_{I5} \triangle \omega_1; \quad x_{I6} \triangle \omega_2; \quad x_{I7} \triangle \omega_3;$$

$$x_{I8} \triangle G_1; \quad x_{I9} \triangle G_2; \quad x_{I10} \triangle G_3;$$

$$u_{I1} \triangle \dot{G}_1; \quad u_{I2} \triangle \dot{G}_2; \quad u_{I3} \triangle \dot{G}_3 .$$

The interconnecting variables from the second subsystem to reflect the coupling between the subsystems are defined as

$$z_{I1} \triangle x_{II1}; \quad z_{I2} \triangle x_{II2}; \quad z_{I3} \triangle x_{II3}; \quad z_{I4} \triangle x_{II4}; \quad z_{I5} \triangle 0;$$

$$z_{I6} \triangle 0; \quad z_{I7} \triangle 0;$$

$$z_{I8} \triangle 0; \quad z_{I9} \triangle 0;$$

$$z_{I10} \triangle 0 .$$

The following state equations are obtained for subsystem I using these definitions

$$\begin{bmatrix} \dot{x}_{I1} \\ \dot{x}_{I2} \\ \dot{x}_{I3} \\ \dot{x}_{I4} \\ \dot{x}_{I5} \\ \dot{x}_{I6} \\ \dot{x}_{I7} \\ \dot{x}_{I8} \\ \dot{x}_{I9} \\ \dot{x}_{I10} \end{bmatrix} = \begin{bmatrix} 0 & D & 0 \\ 0 & 0 & H \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{I1} \\ x_{I2} \\ x_{I3} \\ x_{I4} \\ x_{I5} \\ x_{I6} \\ x_{I7} \\ x_{I8} \\ x_{I9} \\ x_{I10} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_2 & B_3 \\ I & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{I1} \\ u_{I2} \\ u_{I3} \\ z_{I1} \\ z_{I2} \\ z_{I3} \\ z_{I4} \end{bmatrix} + \begin{bmatrix} 0 \\ H \\ 0 \end{bmatrix} N_2 (x_{I1}, x_{I2}, x_{I3}, x_{I4}, x_{I5}, x_{I6}, x_{I7}) \quad (16)$$

$$\text{Here, } H = I_o^{-1} \left[A_2 A^{-1} A_2^T I_o^{-1} + I_3 \right],$$

$$B_2 = I_o^{-1} A_2 A^{-1} K,$$

$$B_3 = I_o^{-1} A_2 A^{-1} B,$$

$$D = \begin{bmatrix} x_{I4} & -x_{I3} & x_{I2} \\ x_{I3} & x_{I4} & -x_{I1} \\ -x_{I2} & x_{I1} & x_{I4} \\ -x_{I1} & -x_{I2} & -x_{I3} \end{bmatrix}.$$

For the second subsystem which is the flexible appendage of the entire system, the first two flexible modes are considered and the corresponding state variables and control variables are defined as

$$x_{II1} \triangle q_1; \quad x_{II2} \triangle q_2;$$

$$x_{II3} \triangle \dot{q}_1; \quad x_{II4} \triangle \dot{q}_2;$$

$$u_{II1} \triangle Q_1; \quad u_{II2} \triangle Q_2; \quad .$$

As in the previous case, the coupling between the two subsystems is derived in terms of the following interconnecting variables from the first subsystem.

$$z_{II1} \triangle x_{I1}; \quad z_{II2} \triangle x_{I2}; \quad z_{II3} \triangle x_{I3};$$

$$z_{II4} \triangle x_{I4};$$

$$z_{II5} \triangle x_{I5}; \quad z_{II6} \triangle x_{I6}; \quad z_{II7} \triangle x_{I7};$$

$$z_{II7} \triangle x_{I8}; \quad z_{II9} \triangle x_{I9}; \quad z_{II10} \triangle x_{I10} .$$

The following are the state equations of this subsystem.

$$\begin{aligned} \begin{bmatrix} \dot{x}_{II1} \\ \dot{x}_{II2} \\ \dot{x}_{II3} \\ \dot{x}_{II4} \end{bmatrix} &= \begin{bmatrix} 0 & I \\ -A^{-1}K & -A^{-1}B \end{bmatrix} \begin{bmatrix} x_{II1} \\ x_{II2} \\ x_{II3} \\ x_{II4} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ A^{-1} & -A^{-1}A_2^T I_o^{-1} \end{bmatrix} \begin{bmatrix} u_{II1} \\ u_{II2} \\ z_{II8} \\ z_{II9} \\ z_{II10} \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ -A^{-1}A_2^T I_o^{-1} \end{bmatrix} N_2(z_{II1}, z_{II2}, z_{II3}, z_{II4}, z_{II5}, z_{II6}, z_{II7}) \end{aligned} \quad (17)$$

The Optimal Control Problem

A general problem for the optimal control of interconnected dynamical systems like large flexible spacecrafts can be formulated as

$$\text{Minimize } J(\underline{x}_i, \underline{u}_i, \underline{z}_i) \quad i=1,2,\dots,N \quad (18)$$

w.r.t. \underline{u}_i

where \underline{x}_i is the n_i dimensional state vector of the i th subsystem, \underline{u}_i is the corresponding m_i dimensional control vector and \underline{z}_i is the r_i dimensional vector of interconnection inputs from the other subsystem. The integer N represents the total number of subsystems and the scalar functional J is defined by

$$J = \sum_{i=1}^N \left\{ P_i (\underline{x}_i(t_f)) + \int_{t_0}^{t_f} L_i \left[\underline{x}_i(t), \underline{u}_i(t), \underline{z}_i(t) \right] dt \right\} \quad (19)$$

where $L_i \left[\underline{x}_i(t), \underline{u}_i(t), \underline{z}_i(t) \right]$ is the performance index at time t for $i = 1, 2, \dots, N$ subsystems. The functional J defined in equation (19) is to be minimized subject to the constraints which define the subsystem dynamics, i.e.

$$\dot{\underline{x}}_i = \underline{f}_i \left[\underline{x}_i(t), \underline{u}_i(t), \underline{z}_i(t), t \right], t_0 \leq t \leq t_f \quad (20)$$

$$\underline{x}_i(t_0) = \underline{x}_{i0}, \quad i = 1, 2, \dots, N.$$

Also, the minimum of J must satisfy the interconnection relationship

$$\sum_{i=1}^N \underline{G}_i^* (\underline{x}_i, \underline{z}_j) = \underline{0}. \quad (21)$$

The Open-loop Hierarchical Control

Using the method of Goal Coordination or infeasible method [15,20], we consider another problem which is obtained by maximizing the dual function $\Phi(\underline{\lambda})$ with respect to $\underline{\lambda}(t)$ ($t_0 \leq t \leq t_f$), where

$$\Phi(\underline{\lambda}) = \min_{\underline{x}, \underline{u}, \underline{z}} \left\{ \tilde{J}(\underline{x}, \underline{u}, \underline{z}, \underline{\lambda}) \right\} \quad (22)$$

subject to constraints in equations (20) and (21). Here

$$\underline{x} = \begin{bmatrix} \underline{x}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{x}_N \end{bmatrix} \quad \underline{u} = \begin{bmatrix} \underline{u}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{u}_N \end{bmatrix} \quad \underline{z} = \begin{bmatrix} \underline{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{z}_N \end{bmatrix} \quad (23)$$

Also, $\underline{\lambda}$ in equation (22) is a vector of Lagrange multipliers which is given as

$$\underline{\lambda} = \begin{bmatrix} \underline{\lambda}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{\lambda}_N \end{bmatrix}. \quad (24)$$

$$\tilde{J} = \sum_{i=1}^N \left\{ P_i (\underline{x}_i(t_f)) + \int_{t_0}^{t_f} L_i (\underline{x}_i, \underline{u}_i, \underline{z}_i, t) dt + \int_{t_0}^{t_f} \underline{\lambda}^T \underline{G}_i^* (\underline{x}_i, \underline{z}_i, t) dt \right\} \quad (25)$$

$j = 1, 2, \dots, N$

Rewriting this functional \tilde{J} as

$$\begin{aligned} \tilde{J} &= \sum_{i=1}^N J_i \\ &= \sum_{i=1}^N \left\{ P_i (\underline{x}_i(t_f)) + \int_{t_0}^{t_f} \left[L_i (\underline{x}_i, \underline{u}_i, \underline{z}_i, t) + \underline{\lambda}^T \underline{G}_i (\underline{x}_i, \underline{z}_i, t) \right] dt \right\} \end{aligned} \quad (26)$$

where,

$$J_i = P_i (\underline{x}_i(t_f)) + \int_{t_0}^{t_f} \left[L_i (\underline{x}_i, \underline{u}_i, \underline{z}_i, t) + \underline{\lambda}^T \underline{G}_i (\underline{x}_i, \underline{z}_i, t) \right] dt \quad (27)$$

and where $\underline{\lambda}^T \underline{G}_i^* (\underline{x}_i, \underline{z}_i, t)$ has been refactored into the form $\underline{\lambda}^T \underline{G}_i (\underline{x}_i, \underline{z}_i, t)$, i.e.

into a form separable in the index i .

Thus,

$$\tilde{J} = J + \int_{t_0}^{t_f} \underline{\lambda}^T \sum_{i=1}^N \underline{G}_i(\underline{x}_i, \underline{z}_i) dt \quad (28)$$

Then by the fundamental theorem of strong Lagrange duality [22]

$$\min_{\underline{u}_i} J = \max_{\underline{\lambda}} \Phi(\underline{\lambda}), \quad i = 1, 2, \dots, N. \quad (29)$$

Thus an alternative way of optimizing J is to maximize $\Phi(\underline{\lambda})$.

From equation (26), for a given $\underline{\lambda}(t)$, $t_0 \leq t \leq t_f$, the functional \tilde{J} is separable into N independent minimization problems, the i th of which is given by

$$\begin{aligned} \text{Min}_{\underline{x}_i, \underline{u}_i, \underline{z}_i} \quad J_i = P_i(\underline{x}_i(t_f)) + \int_{t_0}^{t_f} \left[L_i(\underline{x}_i, \underline{u}_i, \underline{z}_i) + \underline{\lambda}^T \underline{G}_i(\underline{x}_i, \underline{z}_i) \right] dt \end{aligned} \quad (30)$$

subject to

$$\begin{aligned} \dot{\underline{x}}_i &= \underline{f}_i(\underline{x}_i, \underline{u}_i, \underline{z}_i), \quad t_0 \leq t \leq t_f \\ \underline{x}_i(t_0) &= \underline{x}_{i0} \end{aligned} \quad (31)$$

This leads to a two-level optimization structure where on the first level, for given $\underline{\lambda}$, the N independent minimization problems described in equations (30) and (31) are solved and on the second level, the $\underline{\lambda}(t)$ ($t_0 \leq t \leq t_f$) trajectory is improved by an optimization scheme like the steepest ascent method, i.e. from iteration j to $j+1$

$$\underline{\lambda}(t)^{j+1} = \underline{\lambda}(t)^j + \alpha^j + \underline{d}^j(t) \quad t_o \leq t \leq t_f \quad (32)$$

where

$$\underline{d}^j = \nabla \Phi(\underline{\lambda}(t)) = \sum_{i=1}^N \underline{G}_i(\underline{x}_i, \underline{z}_i), \quad (33)$$

$\nabla \Phi(\underline{\lambda})$ is the gradient of $\Phi(\underline{\lambda})$, $\alpha_j > 0$ is the step length and \underline{d}^j is the steepest ascent search direction. At the optimum $\underline{d}^j \rightarrow 0$ and the appropriate Lagrange multiplier, $\underline{\lambda}$, is the optimum one.

The development of this algorithm depends on the assertion $\text{Max } \Phi(\underline{\lambda}) = \text{min } J$ and this may not be valid for all nonlinear systems. Consequently, linearization of \underline{G}_i , and linearized equations for \underline{f}_i may be required for constraints to be convex and convexity of the constraints is necessary to prove this assertion. Nevertheless, the method is attractive from the standpoint of simplicity and that the dual function is concave for this nonlinear case. This ensures that if the duality assertion is valid, the optimum obtained is the Global Optimum.

On the first level, since equation (30) is to be minimized subject to equation (31), the necessary conditions lead to a two point boundary value problem from which an open loop optimum control could be calculated. However, it is desirable to calculate a closed loop control and for this the quasilinearization approach can be utilized at level one for all subsystems. Thus an iterative scheme can be set up whereby an initial trajectory of $\underline{\lambda}(t)^*$, $t_o \leq t \leq t_f$ is guessed at level two and provided to level one. At level one the two-point boundary value problems of the subsystems are solved by quasilinearization. The state and control trajectories of all the subsystems obtained at level one are sent to level two. The test for optimality based on equation (33) is conducted at level two and if this is not satisfied, equation (32) is used to obtain the new $\underline{\lambda}(t)$ for the next iteration.

Subsystem Closed Loop Controllers

The closed loop controllers are obtained at the first level by solving the two-point boundary value problems of the subsystems utilizing the quasilinearization procedure. As noted in equations (30) and (31), the first level problem for the i th subsystem is

For given $\underline{\lambda}(t)$, $t_0 \leq t \leq t_f$,

$$\min_{\underline{x}_i, \underline{u}_i, \underline{z}_i} \left\{ P_i \left[\underline{x}_i(t_f) \right] + \int_{t_0}^{t_f} \left[L_i(\underline{x}_i, \underline{u}_i, \underline{z}_i) + \underline{\lambda}^T \underline{G}_i(\underline{x}_i, \underline{z}_i) \right] dt \right\} \quad (30)$$

subject to

$$\begin{aligned} \dot{\underline{x}}_i &= \underline{f}_i(\underline{x}_i, \underline{u}_i, \underline{z}_i), \quad t_0 \leq t \leq t_f \\ \underline{x}_i(t_0) &= \underline{x}_{i0} \end{aligned} \quad (31)$$

For this problem, the Hamiltonian H_i can be written as

$$H_i = L_i(\underline{x}_i, \underline{u}_i, \underline{z}_i) + \underline{\lambda}^T \underline{G}_i(\underline{x}_i, \underline{z}_i) + \eta_i^T \underline{f}_i(\underline{x}_i, \underline{u}_i, \underline{z}_i). \quad (34)$$

For a given $\underline{\lambda}$, the state and costate equations become

$$\dot{\underline{x}}_i(t) = \underline{f}_i(\underline{x}_i, \underline{u}_i, \underline{z}_i) \quad (35)$$

$$\dot{\underline{\eta}}_i(t) = - \frac{\partial H_i}{\partial \underline{x}_i} = - \left[\frac{\partial L_i}{\partial \underline{x}_i} + \frac{\partial \underline{G}_i^T}{\partial \underline{x}_i} \underline{\lambda} + \frac{\partial \underline{f}_i^T}{\partial \underline{x}_i} \underline{\eta}_i \right] \quad (36)$$

with

$$\frac{\partial H_i}{\partial \underline{u}_i} = \underline{0}; \quad \frac{\partial H_i}{\partial \underline{z}_i} = \underline{0} \quad (37)$$

It is assumed here that using the equations (36) and (37), it is possible to obtain the control \underline{u}_i and the interconnect variable vector \underline{z}_i which is an explicit

function of \underline{n}_i and \underline{x}_i , i.e.

$$\underline{u}_i = \underline{c}_i (\underline{x}_i, \underline{n}_i) \quad (38)$$

$$\underline{z}_i = \underline{d}_i (\underline{x}_i, \underline{n}_i)$$

Using these relationships for \underline{u}_i and \underline{z}_i in equations (35) and (36), the following equations are obtained

$$\dot{\underline{x}}_i = \underline{a}_i (\underline{x}_i, \underline{n}_i) , \quad t_o \leq t \leq t_f \quad (39)$$

$$\dot{\underline{n}}_i = \underline{b}_i (\underline{x}_i, \underline{n}_i) , \quad t_o \leq t \leq t_f \quad (40)$$

with the boundary conditions

$$\underline{x}_i (t_o) = \underline{x}_{io} \quad (41)$$

and from the transversality conditions

$$\underline{n}_i (t_f) = \frac{\partial P_i \left[\underline{x}_i (t_f) \right]}{\partial \underline{x}_i} . \quad (42)$$

Quasilinearization Procedure

The two-point boundary value problem of i th subsystem is given by equations (39) and (40) subject to boundary conditions of equations (41) and (42). This problem is solved by quasilinearization technique as follows.

$$\text{Defining } \underline{y} = \begin{bmatrix} \underline{x}_i \\ \underline{n}_i \end{bmatrix} ,$$

equations (39) and (40) can be rewritten as

$$\dot{\underline{y}} (t) = \underline{F} \left[\underline{y} (t) \right] . \quad (43)$$

In the quasilinearization procedure, starting from an initial guessed trajectory for $\underline{y} = \underline{y}^j(t)$, successive linearizations are performed of equation (43) in such a way that the final linear equation for \underline{y} solves equation (43) to an acceptable degree subject to boundary conditions (41) and (42) which could be expressed in a more general form as

$$\underline{y}(t_o)^T A_o = \underline{b}_o^T \quad (44)$$

$$\underline{y}(t_f)^T A_f = \underline{b}_f^T \quad (45)$$

where A_o, A_f are $2n \times n$ matrices.

The linearized equation of (43) about a trajectory $\underline{y} = \underline{y}^j(t)$ is obtained by Taylor series expansion as

$$\dot{\underline{y}} = \underline{F}(\underline{y}^j) + J(\underline{y}^j)(\underline{y} - \underline{y}^j) + \underline{\Psi} \quad (46)$$

where $J(\underline{y}^j)$ is the Jacobian of $\underline{F}[\underline{y}(t)]$, $t_o \leq t \leq t_f$, at \underline{y}^j and $\underline{\Psi}$ represents the contribution of the higher order terms. Neglecting these higher order terms, the following linear equation is obtained

$$\dot{\underline{y}} = \underline{F}(\underline{y}^j) + J(\underline{y}^j)(\underline{y} - \underline{y}^j) \quad (47)$$

If the initial guessed trajectory \underline{y}^j while satisfying equations (44),(45) and (47) does not satisfy equation (43), then an iterative search can be utilized to obtain a better linearizing trajectory by various methods discussed in references 19, 20, and 21. This iterative search is given by noting that equation (47) can be written by expanding individual equations (39) and (40) by Taylor series expansion about a known trajectory $\underline{x}^j(t), \underline{u}^j(t), t \in [t_o, t_f]$, and retaining terms of up to first order. The linearized reduced differential equations are

$$\begin{aligned} \dot{\underline{x}}_i^{(j+1)} = & \underline{a}_i \left(\underline{x}_i^j(t), \underline{n}_i^j(t) \right) + \left[\frac{\partial \underline{a}_i}{\partial \underline{x}_i} \left(\underline{x}_i^j(t), \underline{n}_i^j(t) \right) \right] \left[\underline{x}_i^{(j+1)}(t) - \underline{x}_i^j(t) \right] \\ & + \left[\frac{\partial \underline{a}_i}{\partial \underline{n}_i} \left(\underline{x}_i^j(t), \underline{n}_i^j(t) \right) \right] \left[\underline{n}_i^{(j+1)}(t) - \underline{n}_i^j(t) \right] \end{aligned} \quad (48)$$

$$\begin{aligned} \dot{\underline{n}}_i^{(j+1)} = & \underline{b}_i \left(\underline{x}_i^j(t), \underline{n}_i^j(t) \right) + \left[\frac{\partial \underline{b}_i}{\partial \underline{x}_i} \left(\underline{x}_i^j(t), \underline{n}_i^j(t) \right) \right] \left[\underline{x}_i^{(j+1)}(t) - \underline{x}_i^j(t) \right] \\ & + \left[\frac{\partial \underline{b}_i}{\partial \underline{n}_i} \left(\underline{x}_i^j(t), \underline{n}_i^j(t) \right) \right] \left[\underline{n}_i^{(j+1)}(t) - \underline{n}_i^j(t) \right] \end{aligned} \quad (49)$$

These differential equations can be rewritten as

$$\dot{\underline{x}}_i^{(j+1)} = A_{11}(t) \underline{x}_i^{(j+1)}(t) + A_{12}(t) \underline{n}_i^{(j+1)}(t) + \underline{e}_1^j(t) \quad (50)$$

$$\dot{\underline{n}}_i^{(j+1)} = A_{21}(t) \underline{x}_i^{(j+1)}(t) + A_{22}(t) \underline{n}_i^{(j+1)}(t) + \underline{e}_2^j(t) \quad (51)$$

or, in the partitioned matrix form,

$$\begin{bmatrix} \dot{\underline{x}}_i^{(j+1)}(t) \\ \dot{\underline{n}}_i^{(j+1)}(t) \end{bmatrix} = \begin{bmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{bmatrix} \begin{bmatrix} \underline{x}_i^{(j+1)}(t) \\ \underline{n}_i^{(j+1)}(t) \end{bmatrix} + \begin{bmatrix} \underline{e}_1^j(t) \\ \underline{e}_2^j(t) \end{bmatrix} \quad (52)$$

where the matrices

$$A_{11}(t) \triangleq \frac{\partial \underline{a}_i}{\partial \underline{x}_i}, \quad A_{12}(t) \triangleq \frac{\partial \underline{a}_i}{\partial \underline{n}_i},$$

$$A_{21}(t) \triangleq \frac{\partial \underline{b}_i}{\partial \underline{x}_i}, \quad A_{22}(t) \triangleq \frac{\partial \underline{b}_i}{\partial \underline{n}_i},$$

and

$$\underline{e}_1^j \triangleq -A_{11}(t) \underline{x}_i^j(t) - A_{12}(t) \underline{n}_i^j(t) + \underline{a}_i \quad (53)$$

$$\underline{e}_2^j \triangleq -A_{21}(t) \underline{x}_i^j(t) - A_{22}(t) \underline{n}_i^j(t) + \underline{b}_i$$

are evaluated at $\underline{x}_i^j(t)$, $\underline{n}_i^j(t)$ and hence are known functions of time.

The method of complementary functions [24] can be incorporated with this linearization of the differential equations in the implementation of iterative search.

An initial guess, $\underline{x}_i^0, \underline{n}_i^0, t \in [t_0, t_f]$, is used to evaluate matrices in equations (53) at the beginning of the first iteration. In the next step, n sets of solutions to the $2n$ homogeneous differential equations

$$\dot{\underline{x}}_i^{(j+1)} = A_{11}(t) \underline{x}_i^{(j+1)}(t) + A_{12}(t) \underline{n}_i^{(j+1)}(t) \quad (54)$$

$$\dot{\underline{n}}_i^{(j+1)} = A_{21}(t) \underline{x}_i^{(j+1)}(t) + A_{22}(t) \underline{n}_i^{(j+1)}(t) \quad (55)$$

are generated by numerical integration. For $(j+1)$ st iteration, these solutions are denoted by $\underline{x}_i^{H1}, \underline{n}_i^{H1}; \underline{x}_i^{H2}, \underline{n}_i^{H2}; \dots; \underline{x}_i^{Hn}, \underline{n}_i^{Hn}$. The boundary conditions used in generating these solutions are

$$\begin{aligned} \underline{x}_i^{H1}(t_0) &= \underline{0}, & \underline{n}_i^{H1}(t_0) &= \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix}^T, \\ \underline{x}_i^{H2}(t_0) &= \underline{0}, & \underline{n}_i^{H2}(t_0) &= \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \end{bmatrix}^T, \\ & \vdots & & \vdots \\ \underline{x}_i^{Hn}(t_0) &= \underline{0}, & \underline{n}_i^{Hn}(t_0) &= \begin{bmatrix} 0 & 0 & 0 & \dots & 1 \end{bmatrix}^T. \end{aligned} \quad (56)$$

Next, one particular solution at $(j+1)$ denoted by $\underline{x}_i^P, \underline{n}_i^P$, is generated by numerically integrating equation (52) from t_0 , to t_f , using the boundary conditions $\underline{x}_i^P(t_0) = \underline{x}_0, \underline{n}_i^P(t_0) = \underline{0}$. Then, the complete solution of equation (52) can be obtained by using the principle of superposition and is of the form

$$\underline{x}_i^{(j+1)}(t) = c_1 \underline{x}_i^{H1}(t) + c_2 \underline{x}_i^{H2}(t) + \dots + c_n \underline{x}_i^{Hn}(t) + \underline{x}_i^P(t) \quad (57)$$

$$\underline{n}_i^{(j+1)}(t) = c_1 \underline{n}_i^{H1}(t) + c_2 \underline{n}_i^{H2}(t) + \dots + c_n \underline{n}_i^{Hn}(t) + \underline{n}_i^P(t) \quad (58)$$

where the values of c_1, c_2, \dots, c_n which make $\underline{n}_i^{(j+1)}(t_f) = \underline{n}_f$ are to be determined. To find these values of c_1, c_2, \dots, c_n , we let $t = t_f$ in equation

(58) and write it as

$$\underline{n}_f = \left[\underline{n}_i^{H1}(t_f) \quad \underline{n}_i^{H2}(t_f) \dots \underline{n}_i^{Hn}(t_f) \right]^{-1} \underline{c} + \underline{n}_i^p(t_f). \quad (59)$$

Here, $\underline{c} \triangleq [c_1 \ c_2 \ \dots \ c_n]^T$ is unknown. Solving for \underline{c} yields

$$\underline{c} = \left[\underline{n}_i^{H1}(t_f) \quad \underline{n}_i^{H2}(t_f) \dots \underline{n}_i^{Hn}(t_f) \right]^{-1} \left[\underline{n}_f - \underline{n}_i^p \right]. \quad (60)$$

It is important to note that the indicated matrix inversion in equation (60) has to exist in order to solve for \underline{c} . Substituting this solution of \underline{c} into equations (57) and (58) gives the $(j+1)$ st trajectory. This completes one iteration of the quasilinearization algorithm and this trajectory can be further utilized to begin another iteration, if required. Generally, the iterative scheme is terminated by comparing the j th and $j+1$ st trajectories by calculating the norm shown in the following equation and comparing it with a preselected termination constant, ρ .

$$\left\| \begin{bmatrix} \underline{x}_i^{(j+1)} \\ \underline{H}^{(j+1)} \\ \underline{n}_i^{(j+1)} \end{bmatrix} - \begin{bmatrix} \underline{x}_i^j \\ \underline{H}^j \\ \underline{n}_i^j \end{bmatrix} \right\| \leq \rho \quad (61)$$

Closed Loop Control

In order to obtain the closed loop control, the solution of the linearized equation (47) can be written as

$$\underline{y}(t_f) = \phi(t_f, t) \underline{y}(t) + \int_t^{t_f} \phi(t_f, \tau) \left[\underline{E}(\tau) - J \underline{y}(\tau) \right] d\tau \quad (62)$$

where ϕ is the state transition matrix of the system in equation (47). Rewriting equation (62) in terms of solutions of states and costates and replacing the integral terms by $\underline{p}_i(t)$

$$\begin{bmatrix} \underline{x}_i(t_f) \\ \underline{n}_i(t_f) \end{bmatrix} = \begin{bmatrix} \phi_{11}(t_f, t) & \phi_{12}(t_f, t) \\ \phi_{21}(t_f, t) & \phi_{22}(t_f, t) \end{bmatrix} \begin{bmatrix} \underline{x}_i(t) \\ \underline{n}_i(t) \end{bmatrix} + \begin{bmatrix} \underline{p}_{i1}(t) \\ \underline{p}_{i2}(t) \end{bmatrix} . \quad (63)$$

From equations (42) and (63)

$$\underline{n}_i(t_f) = \frac{\partial P_i}{\partial \underline{x}_i} = \phi_{21}(t_f, t) \underline{x}_i(t_f, t) + \phi_{22}(t_f, t) \underline{n}_i(t) + \underline{p}_{i2}(t) . \quad (64)$$

Thus,

$$\begin{aligned} \underline{n}_i(t) &= \phi_{22}^{-1} \left[\frac{\partial P_i}{\partial \underline{x}_i} - \phi_{21} \underline{x}_i(t) - \underline{p}_{i2}(t) \right] \\ &= \phi_{22}^{-1} \left[\frac{\partial P_i}{\partial \underline{x}_i} - \underline{p}_{i2}(t) \right] - \phi_{22}^{-1} \left[\phi_{21}(t) \right] . \end{aligned} \quad (65)$$

It is important to note here that ϕ_{22}^{-1} always exists since it is a principal minor of the state transition matrix.

Substituting equation (65) into equation (38)

$$\underline{u}_i = \underline{c}_i(\underline{x}_i, t) . \quad (66)$$

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APPENDIX A

The nonlinear term \underline{N}_2 is given in terms of the attitude of the orbiter, $\underline{\theta}$, and angular velocity vector transformation matrix, M , as

$$\underline{N}_2 = M^{-1} \begin{bmatrix} \underline{\omega}^T M^{-1} \frac{\partial M}{\partial \theta_1} \\ \underline{\omega}^T M^{-1} \frac{\partial M}{\partial \theta_2} \\ \underline{\omega}^T M^{-1} \frac{\partial M}{\partial \theta_3} \end{bmatrix} - \dot{M} \left[I_o \underline{\omega} + A_2 \dot{\underline{\theta}} \right] \quad (A-1)$$

where

$$M = \begin{bmatrix} \cos\theta_2 \cos\theta_3 & -\cos\theta_2 \sin\theta_3 & \sin\theta_2 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (A-2)$$

$$\underline{\omega} = M^T \dot{\underline{\theta}} \quad (A-3)$$

and

$$\underline{\omega}^T M^{-1} \frac{\partial M}{\partial \theta_1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad (A-4)$$

$$\underline{\omega}^T M^{-1} \frac{\partial M}{\partial \theta_2} = \frac{1}{\cos\theta_2} \begin{bmatrix} (-\omega_1 \sin\theta_2 \cos^2\theta_3 + \omega_2 \sin\theta_2 \sin\theta_3 \cos\theta_3) & (\omega_1 \sin\theta_2 \sin\theta_3 \cos\theta_3 \\ -\omega_2 \sin\theta_2 \sin^2\theta_3) & (\omega_1 \cos\theta_2 \cos\theta_3 - \omega_2 \cos\theta_2 \sin\theta_3) \end{bmatrix} \quad (A-5)$$

$$\underline{\omega}^T M^{-1} \frac{\partial M}{\partial \theta_3} = \frac{1}{\cos\theta_2} \begin{bmatrix} (\omega_2 \cos\theta_2) & (-\omega_1 \cos\theta_2) & 0 \end{bmatrix}. \quad (A-6)$$

Since the transformation matrix, M , is a function of θ_2 and θ_3 , the time derivative of M can be expressed by the chain rule as

$$\dot{M} = \frac{\partial M}{\partial \theta_2} \dot{\theta}_2 + \frac{\partial M}{\partial \theta_3} \dot{\theta}_3 \quad . \quad (A-7)$$

From equation (A-1)

$$\frac{\partial M}{\partial \theta_2} \dot{\theta}_2 = \begin{vmatrix} (-\sin \theta_2 \cos \theta_3) \dot{\theta}_2 & (\sin \theta_2 \sin \theta_3) \dot{\theta}_2 & (\cos \theta_2) \dot{\theta}_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \quad (A-8)$$

$$\frac{\partial M}{\partial \theta_3} \dot{\theta}_3 = \begin{vmatrix} (-\cos \theta_2 \sin \theta_3) \dot{\theta}_3 & (-\cos \theta_2 \cos \theta_3) \dot{\theta}_3 & 0 \\ (\cos \theta_3) \dot{\theta}_3 & (-\sin \theta_3) \dot{\theta}_3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \quad . \quad (A-9)$$

Substituting these equations (A-8) and (A-9) in (A-7)

$$\dot{M} = \begin{vmatrix} (-\sin \theta_2 \cos \theta_3) \dot{\theta}_2 + (-\cos \theta_2 \sin \theta_3) \dot{\theta}_3 & (\sin \theta_2 \sin \theta_3) \dot{\theta}_2 + (-\cos \theta_2 \cos \theta_3) \dot{\theta}_3 & (\cos \theta_2) \dot{\theta}_2 \\ (\cos \theta_3) \dot{\theta}_3 & (-\sin \theta_3) \dot{\theta}_3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \quad (A-10)$$

From equation (A-3), this can also be expressed as

$$\dot{M} = \frac{1}{\cos \theta_2} \begin{vmatrix} (-\sin \theta_2 \cos \theta_3)(\omega_1 \cos \theta_2 \sin \theta_3 + \omega_2 \cos \theta_2 \cos \theta_3) + (-\cos \theta_2 \sin \theta_3)(-\omega_1 \sin \theta_2 \cos \theta_3 + \omega_2 \sin \theta_2 \sin \theta_3 + \omega_3 \cos \theta_2) & (\sin \theta_2 \sin \theta_3)(\omega_1 \cos \theta_2 \sin \theta_3 + \omega_2 \cos \theta_2 \cos \theta_3) + (-\cos \theta_2 \cos \theta_3)(-\omega_1 \sin \theta_2 \cos \theta_3 + \omega_2 \sin \theta_2 \sin \theta_3 + \omega_3 \cos \theta_2) & 0 \\ (\cos \theta_3)(-\omega_1 \sin \theta_2 \cos \theta_3 + \omega_2 \sin \theta_2 \sin \theta_3 + \omega_3 \cos \theta_2) & (-\sin \theta_3)(-\omega_1 \sin \theta_2 \cos \theta_3 + \omega_2 \sin \theta_2 \sin \theta_3 + \omega_3 \cos \theta_2) & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\begin{bmatrix} \cos\theta_2(\omega_1\cos\theta_2\sin\theta_3+\omega_2\cos\theta_2\cos\theta_3) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (A-11)$$

Also, M^{-1} is given as

$$M^{-1} = \frac{1}{\cos\theta_2} \begin{bmatrix} \cos\theta_3 & \cos\theta_2\sin\theta_3 & -\sin\theta_2\cos\theta_3 \\ -\sin\theta_3 & \cos\theta_2\cos\theta_3 & \sin\theta_2\sin\theta_3 \\ 0 & 0 & \cos\theta_2 \end{bmatrix} \quad (A-12)$$

Thus, the nonlinear term N_2 can be rewritten as

$$N_2 = A'_3(\underline{\omega}, \underline{\theta}) \left[I_o \underline{\omega} + A_2 \dot{\underline{q}} \right]$$

Where the term A'_3 is

$$A'_3(\underline{\omega}, \underline{\theta}) = M^{-1} \begin{bmatrix} 0 \\ \underline{\omega}^T M^{-1} \frac{\partial M}{\partial \theta_2} \\ \underline{\omega}^T M^{-1} \frac{\partial M}{\partial \theta_3} \end{bmatrix} - \dot{M}$$

$$\begin{aligned} N_2 &= A'_3(\underline{\omega}, \underline{\theta}) I_o \underline{\omega} + A'_3(\underline{\omega}, \underline{\theta}) A_2 \dot{\underline{q}} \\ &= A_4(\underline{\omega}, \underline{\theta}) + A_5(\underline{\omega}, \underline{\theta}) \dot{\underline{q}} \end{aligned} \quad (A-13)$$

where A_4 depends on the rigid-body slewing and is nonlinear in terms of $\underline{\omega}$ and $\underline{\theta}$. The second term relates the coupling between the rigid-body slewing and the flexible modes.

APPENDIX B

The transformation that relates the orientation angles $\underline{\theta}$ to Euler parameters $\underline{\epsilon}$ is a nonlinear transformation. This transformation is developed for body-three angles representation in this appendix and similar transformations can be derived for other three representations, namely space-three angles, space-two angles, and body-two angles.

(a) For $\sin\theta_2 \neq 1$:

If $-\frac{\pi}{2} < \theta_2 < \frac{\pi}{2}$, then

$$\theta_2 = \sin^{-1} \left[2(\epsilon_3 \epsilon_1 + \epsilon_2 \epsilon_4) \right] . \quad (B-1)$$

If $(\cos\theta_1 \cos\theta_2) \geq 0$, then

$$\theta_1 = \sin^{-1} \left[\frac{-2(\epsilon_2 \epsilon_3 - \epsilon_1 \epsilon_4)}{\cos \left[\sin^{-1} \left[2(\epsilon_3 \epsilon_1 + \epsilon_2 \epsilon_4) \right] \right]} \right] . \quad (B-2)$$

If $(\cos\theta_1 \cos\theta_2) < 0$, then

$$\theta_1 = \pi - \sin^{-1} \left[\frac{-2(\epsilon_2 \epsilon_3 - \epsilon_1 \epsilon_4)}{\cos \left[\sin^{-1} \left[2(\epsilon_3 \epsilon_1 + \epsilon_2 \epsilon_4) \right] \right]} \right] . \quad (B-3)$$

If $(\cos\theta_2 \cos\theta_3) \geq 0$, then

$$\theta_3 = \sin^{-1} \left[\frac{-2(\epsilon_1 \epsilon_2 - \epsilon_3 \epsilon_4)}{\cos \left[\sin^{-1} \left[2(\epsilon_3 \epsilon_1 + \epsilon_2 \epsilon_4) \right] \right]} \right] . \quad (B-4)$$

If $(\cos\theta_2 \cos\theta_3) < 0$, then

$$\theta_3 = \pi - \sin^{-1} \left[\frac{-2 (\epsilon_1 \epsilon_2 - \epsilon_3 \epsilon_4)}{\cos \left[\sin^{-1} \left[2 (\epsilon_3 \epsilon_1 + \epsilon_3 \epsilon_2) \right] \right]} \right]. \quad (\text{B-5})$$

(b) For $\sin \theta_2 = \pm 1$, θ_2 is a constant. For $\sin \theta_2 = 1$, $\theta_2 = \frac{\pi}{2}$. However, if $\sin \theta_2 = -1$, then $\theta_2 = -\frac{\pi}{2}$. For this case, if $(\sin \theta_1 \sin \theta_2 \sin \theta_3 + \cos \theta_3 \cos \theta_1) \geq 0$, then

$$\theta_1 = \sin^{-1} \left[2 (\epsilon_2 \epsilon_3 + \epsilon_1 \epsilon_4) \right]. \quad (\text{B-6})$$

If $(\sin \theta_1 \sin \theta_2 \sin \theta_3 + \cos \theta_3 \cos \theta_1) < 0$, then

$$\theta_1 = \pi - \sin^{-1} \left[2 (\epsilon_2 \epsilon_3 + \epsilon_1 \epsilon_4) \right]. \quad (\text{B-7})$$

For this entire case, $\theta_3 = 0$.

APPENDIX C

Numerical Data:

The analytics developed in reference [1] are utilized together with the basic SCOLE data [2] and the three dimensional linear vibration analysis [25] to generate the following numerical data.

$$\underline{r} = \begin{bmatrix} 0.036 \\ -0.036 \\ -0.379 \end{bmatrix}$$

$$I_o = \begin{bmatrix} 1216640 & -1.530307 & 175667.1 \\ -31.66433 & 7082976 & -52474.84 \\ 175690 & -52503.9 & 7131493 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.45879E+2 & 0.36305E-1 & -0.89042E-1 & -0.14067E0 & 0.1457E0 \\ 0.36305E-1 & 0.6211E+2 & 0.11263E0 & -0.1471E0 & -0.5518E-1 \\ -0.89042E-1 & 0.11263E0 & 0.32737E+2 & -0.6392E-1 & -0.14526E0 \\ -0.14067E0 & -0.1471E0 & -0.6392E-1 & 0.2547E+3 & 0.1908E0 \\ -0.1457E0 & -0.5518E-1 & -0.14526E0 & 0.1908E0 & 0.8103E+3 \\ 0.1914E-1 & 0.19839E-1 & 0.7925E-2 & -0.4278E-1 & -0.2570E-1 \\ 0.84597E-1 & 0.3935E-2 & -0.8369E-1 & -0.76115E-1 & -0.12912E0 \\ -0.6893E-2 & -0.7165E-2 & -0.2829E-2 & 0.1543E-1 & 0.9222E-2 \\ -0.4269E-1 & 0.5969E-2 & 0.89767E-1 & 0.2859E-1 & 0.4611E-1 \\ 0.4204E-2 & 0.41227E-2 & 0.1866E-2 & -0.9067E-2 & -0.5947E-2 \end{bmatrix}$$

$$\begin{bmatrix} 0.1914E-1 & 0.84597E-1 & -0.6893E-2 & -0.4269E-1 & 0.4204E-2 \\ 0.19839E-1 & 0.3935E-2 & -0.7165E-2 & 0.5969E-2 & 0.4127E-2 \\ 0.7925E-2 & -0.8369E-1 & -0.2829E-2 & 0.89767E-1 & 0.1866E-2 \\ -0.4278E-1 & -0.76115E-1 & 0.1543E-1 & 0.2859E-1 & -0.9067E-2 \\ -0.2570E-1 & -0.12912E0 & 0.9222E-2 & 0.4611E-1 & -0.5947E-2 \\ 0.23209E+5 & 0.10383E-1 & -0.2089E-2 & -0.3955E-2 & 0.1227E-2 \\ 0.10383E-1 & 0.55561E+5 & -0.37286E-2 & -0.3859E-1 & 0.2397E-2 \\ -0.2089E-2 & -0.37286E-2 & 0.1342962E+8 & 0.1421E-2 & -0.4427E-3 \\ -0.3955E-2 & -0.3859E-1 & 0.1421E-2 & 0.2095672E+8 & -0.9108E-3 \\ 0.1227E-2 & 0.2397E-2 & -0.4427E-3 & -0.9108E-3 & 0.8662547E+10 \end{bmatrix}$$

$$A_2^T = \begin{bmatrix} -0.2133821E0 & -0.3687057E+3 & -0.7253901E-1 \\ 0.3808921E+3 & -0.3030935E+2 & -0.8427658E-1 \\ -0.1808478E+3 & -0.1318596E+3 & -0.125799E0 \\ 0.1423380E+3 & -0.1135851E+1 & -0.2367351E-1 \\ -0.2416743E+2 & 0.574383E+2 & -0.9150328E-1 \\ -0.6802273E0 & 0.3104929E2 & -0.3843062E-1 \\ 0.2784792E+2 & 0.6651585E+2 & 0.596075E-1 \\ 0.7842818E+1 & -0.1930097E+2 & -0.4363533E-2 \\ -0.2694455E+2 & -0.5544252E+2 & -0.4200623E-1 \\ -0.9225328E-1 & 0.1594045E+2 & -0.1626004E-1 \end{bmatrix}$$

The stiffness matrix K is calculated using equation (57) and the mode shape coefficients given in Table 1. This matrix is a diagonal matrix and is represented in terms of the diagonal elements as

$$K = \begin{bmatrix} k_{1,1} = 0.2820217E0 \\ k_{2,2} = 0.3574692E0 \\ k_{3,3} = 0.2412807E1 \\ k_{4,4} = 0.5285116E1 \\ k_{5,5} = 0.1588654E2 \\ k_{6,6} = 0.8573860E2 \\ k_{7,7} = 0.1146118E3 \\ k_{8,8} = 0.5686101E3 \\ k_{9,9} = 0.6254598E3 \\ k_{10,10} = 0.2114612E4 \end{bmatrix}$$

The damping matrix B used for this analysis is a diagonal matrix and for damping ratio $\zeta = 0.003$, it is calculated to be

$$B = \begin{bmatrix} b_{1,1} = 0.9685964E-3 \\ b_{2,2} = 0.1088608E-2 \\ b_{3,3} = 0.2834016E-2 \\ b_{4,4} = 0.4256808E-2 \\ b_{5,5} = 0.7387177E-2 \\ b_{6,6} = 0.1719014E-1 \\ b_{7,7} = 0.1984237E-1 \\ b_{8,8} = 0.4421234E-1 \\ b_{9,9} = 0.4633434E-1 \\ b_{10,10} = 0.8527647E-1 \end{bmatrix}$$

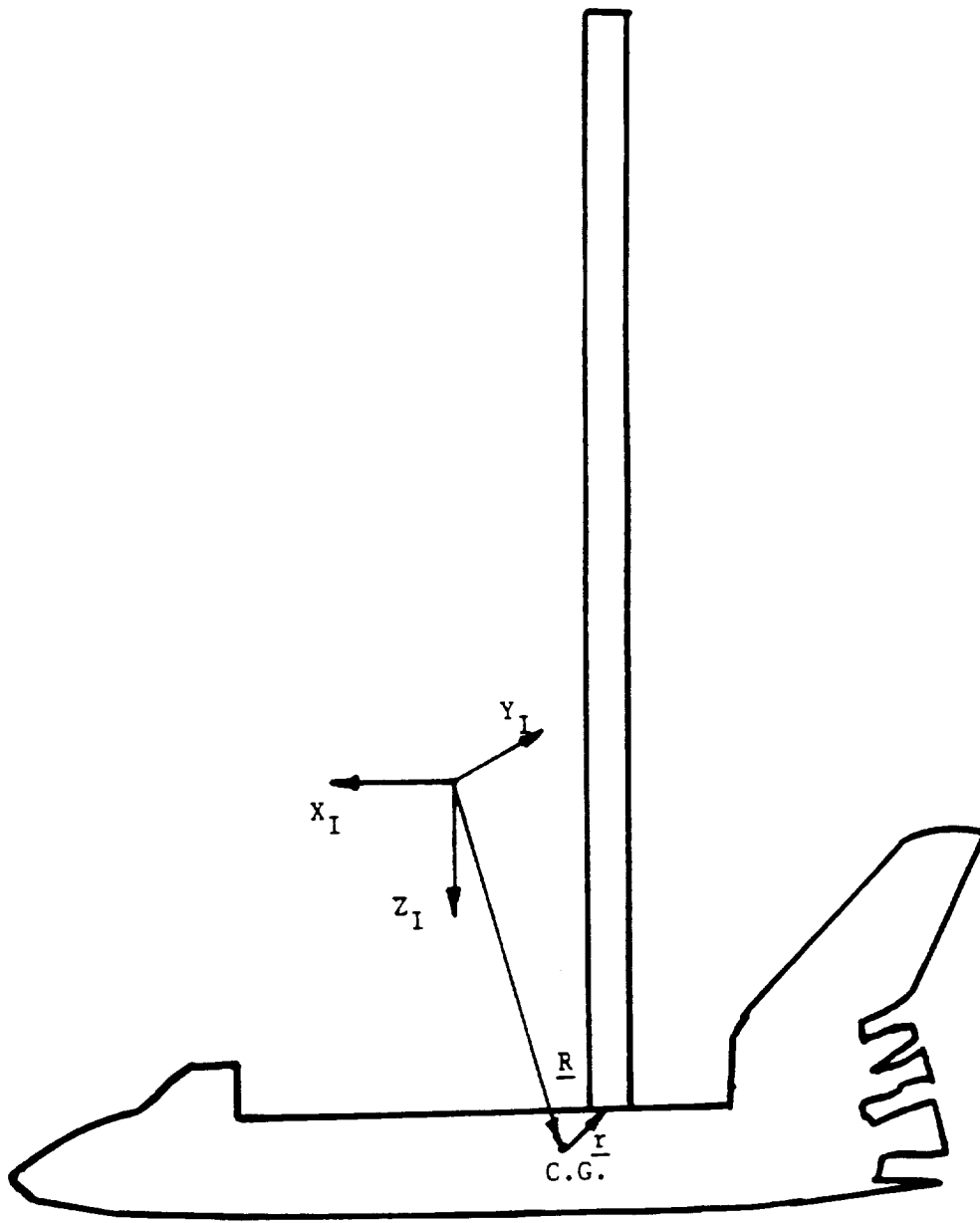


Figure 1- Position Vectors in Inertial Frame

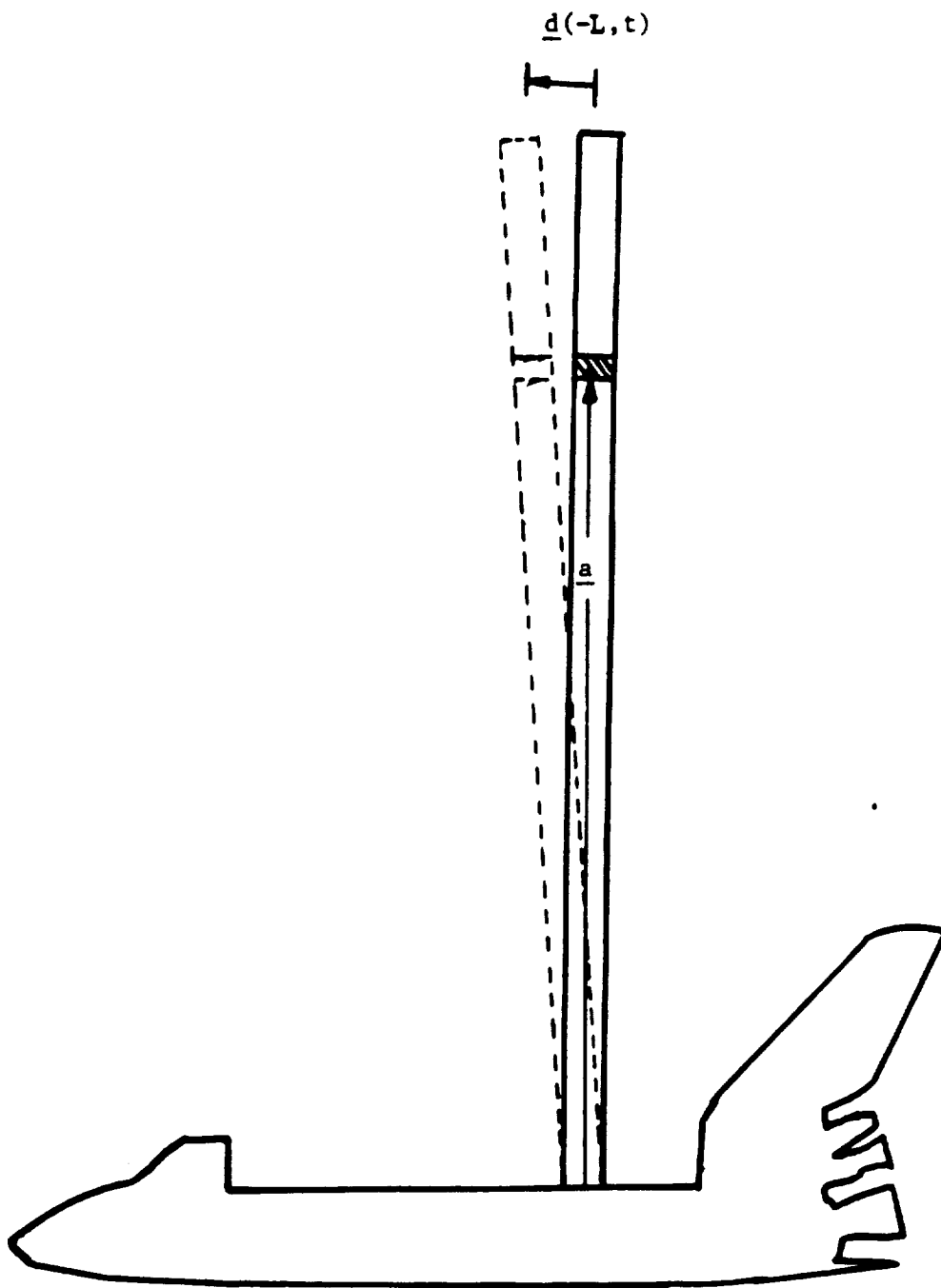


Figure 2- Vectors in Body-fixed Frame